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COOLDOWN AND WARMUP OF LARGE POWDER-INSULATED DEWARS

by

F. Kreith[*,†], L. Brooks[†], and J. W. Dean[*]

Introduction.

Transient heat transfer phenomena related to the use of large storage containers for cryogenic liquid propellants have become increasingly important in rocket and missile technology. This paper presents a theoretical analysis of the heat transfer processes during the cooldown and the warmup periods of such containers. The results can help a designer to select the appropriate type of insulation for ground use or for orbital storage, to predict the transient rate of heat transfer to a container during cooldown after filling and to estimate the time required for warmup of a cryogenic storage container.

Notation.

a	-	inner radius of sphere
b	-	outer radius of sphere
c_i	-	specific heat capacity of the insulation
c_m	-	specific heat capacity of the metal wall
C_n	-	constant
h	-	unit surface conductance
k_i	-	thermal conductivity of the insulation
k_m	-	thermal conductivity of the metal wall
L	-	thickness of the insulation
q	-	rate of heat transfer
T	-	temperature
x	-	perpendicular distance from outer interface of the insulation

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α - thermal diffusivity of the insulation

δ_m - thickness of the inner metal layer of tank wall

λ_n - Eigen value

θ - time

ρ - density

Cooldown

When an insulated storage dewar at ambient temperature is filled with a liquefied gas, considerable time is required before the rate of heat transfer to the liquefied gas reaches equilibrium. Blanks and Timmerhaus (1) have called attention to the fact that although the steady state heat leak can always be reduced by thickening the insulation, the optimum design of a dewar for minimum heat loss must consider the cooldown period because in many practical situations the transfer process never reaches steady state conditions. Stoy (2) has calculated the variation in heat loss during the cooldown period for a large vessel with three different types of insulations, but since he used a numerical method his results lack generality and can not readily be applied to other cases.

Analytical solutions for the temperature response of a two layer series-composite wall, with both materials having finite thermal conductivities, were presented in Refs. 3 and 4 for the case of sudden exposure to a uniform environment at a different temperature through a constant convection heat transfer coefficient at the surface. Numerical results, however, were calculated only for the case where the thermal conductivity of the layer exposed to the heat source was lower than the conductivity of the backup layer whose rear face was considered adiabatic. Eigen values for a two layer series-composite wall in which the inverse arrangement of materials was present, namely a good conductor over an insulator, are presented in Ref. 5. Although the problems treated in Ref. 5 encompasses the case of suddenly cooling the wall of a cryogenic storage container initially at a uniform temperature, the specific case under consideration is amendable to a simpler analysis because the difference in the thermal conductivities between the two layers, namely the metal wall and the insulation, is very large.

During cooldown, heat is transferred from the inner dewar wall, which is initally at ambient temperature, to the liquefied gas. If the dewar is very large and the insulation thickness is small compared with the diameter, the system can be idealized, without introducing an appreciable error, by a two-dimensional slab having the same properties as the insulation. It can be assumed that the thermal resistances between the liquefied gas and the inner surface of the insulation, and also between the outer surface of the insulation and the surroundings, are very small compared to the thermal resistance of the insulation. The temperature history of the insulation can then be described by the equation

$$\rho c_i \frac{\partial T}{\partial \theta} = - \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) \tag{1}$$

subject to the following boundary and initial conditions:

1.) The temperature at the exterior surface (x = 0) is equal to the temperature of the environment which is taken as the datum and set equal to zero, or

$$T(\theta, O) = O \tag{2}$$

2.) The temperature at the inner surface (x = L) is equal to the temperature of the liquefied gas, or

$$T(\theta, L) = -T_o \tag{3}$$

3.) The temperature in the entire insulation ($O \leq x \leq L$) is initially equal to the environmental temperature, or

$$T(O, x) = O \tag{4}$$

Assuming that the effective thermal conductivity of the insulation k and the heat capacity per unit volume of the insulation $c\rho$ are independent of temperature, eq. 1 becomes

$$\frac{\partial T}{\partial \theta} = - \alpha \frac{\partial^2 T}{\partial x^2} \qquad (5)$$

If, on the other hand, only the thermal conductivity is temperature dependent, eq. 1 can be written in the form

$$\frac{\partial T}{\partial \theta} = - \alpha \frac{\partial^2 T}{\partial x^2} - \frac{1}{\rho c_i} \left(\frac{\partial T}{\partial x} \right)^2 \frac{\partial k}{\partial T} \qquad (6)$$

which also reduces to eq. 5 if the temperature coefficient of the thermal conductivity is small compared to the heat capacity per unit volume and the thermal diffusivity is uniform and constant.

An analytical solution of eq. 5 for the boundary conditions describing the system under analysis can be found in Ref. 6. Substituting the initial condition T(O, x) = O into this solution, it can be shown (7) that the following equation gives the temperature distribution at any time θ:

$$T = - T_o \left[\frac{x}{L} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} e^{-(\alpha n^2 \pi^2 \frac{\theta}{L^2})} \sin \frac{n\pi x}{L} \right] \qquad (7)$$

The rate of heat transfer per unit area from the liquefied gas is equal to the rate of heat conduction to the inner surface of the insulation, or

$$\frac{q}{A} = -k_i \frac{\partial T}{\partial x} (\theta, O) = k_i \frac{T_o}{L} \left[1 + 2 \sum_{n=1}^{\infty} e^{-(\alpha n^2 \pi^2 \frac{\theta}{L^2})} \right] \qquad (8)$$

The ratio of the rate of heat transfer at any time θ after filling to the rate of heat transfer in the steady state is obtained by dividing eq. 8 by kT_o/L, or

$$\frac{q(\theta)}{q_{\text{steady state}}} = 1 + 2 \sum_{n=1}^{\infty} e^{-\left(\alpha n^2 \pi^2 \frac{\theta}{L^2}\right)} \tag{9}$$

Fig. 1 shows the ratio of the rate of heat transfer at time θ to the steady state heat loss as a function of the dimensionless Fourier modulus $\alpha\theta/L^2$. Fig. 2 shows $q(\theta)/q$ (steady state) as a function of time for vessels insulated with 30.5 cm ($\approx$ 1 ft.), 61 cm ($\approx$ 2 ft.), and 91.5 cm ($\approx$ 3 ft.) thicknesses of an insulation having a thermal diffusivity of 1.8×10^{-4} $cm^2/sec.$ ($\approx 7 \times 10^{-4}$ $ft^2/hr.$).

For a spherical dewar, an equation for the transient rate of heat transfer can be similarly derived from Ref. 6 and put into the form

$$\frac{q(\theta)}{q_{\text{steady state}}} = 1 + 2\,\frac{a}{b} \sum_{n=1}^{\infty} e^{-\left(\alpha n^2 \pi^2 \frac{\theta}{(b-a)}\right)} \tag{10}$$

A comparison of equations 9 and 10 shows that the effect of the spherical shape is to reduce each term in the series solution for the slab by the factor (a/b). The dotted curve in Fig. 2 shows $q(\theta)/q$ (steady state) as a function of time for a 55,000 gallon spherical tank with an inside diameter of 732 cm ($\approx$ 24 ft.) and an insulation thickness of 91.5 cm ($\approx$ 3 ft.).

If the heat capacity of the inner metal wall of a dewar can not be neglected compared to the heat capacity of the insulation, but the Biot Number ($h\delta/k_m$) of the metal wall is sufficiently large that its own thermal resistance is negligible, it is apparent that equation 7 remains unaffected (8). This situation will generally prevail when the dewar wall consists of powder or multiple layer insulation, sandwiched between two metal sheets. The only additional factor to be considered in this case is the vaporization of liquefied gas which will occur immediately after filling the tank as the inner metal wall approaches the temperature of the liquefied gas. During this short interval an amount of energy equal to the change in internal energy of the inner metal wall of the dewar will be transferred as heat to the liquefied gas and increase its enthalpy approximately by an amount $Ac_m \rho_m \delta_m$ where A is the inner surface area of the entire tank. After the initial vaporization has occurred, eq. 9 or 10 will apply since no further change in wall temperature will take place. However, since in practice the initial evaporation is quite vigorous, the effect of the heat capacity of the wall does not affect the transient analysis for the insulation alone appreciably.

Warmup

After a cryogenic storage container has been emptied, it is often necessary to warm it back up to the temperature of the environment in order to check or install instruments, remove impurities, or make adjustments before refilling. Since most cryogenic vessels are insulated with materials of very low thermal conductivity, the period of warmup may be very long. One method of accelerating the warmup is to break the vacuum in the insulation at least to a point where the thermal conductivity increases sharply as shown in Fig. 6 of Ref. 10. The vacuum can be broken by introducing a variety of gases and in order to select the most appropriate medium it is necessary to investigate the warmup process. The following analysis is designed to make a time estimate of the transient condition.

The basic equation describing the system is the same as for the cooldown. However, the thermal capacity of the inner metal layer is now connected through a very large thermal resistance to the heat reservoir and must be taken into account. An exact solution of the system would require application of eq. 1 to both the inner wall and the insulation of the dewar and connecting these two sub-systems by the first law of thermodynamics which requires continuity of heat flow at the interface between them. A solution to these two simultaneous partial differential equations is theoretically feasible (3, 5), but very complicated. In cryogenic containers however, the thermal conductivity of the metal wall is so much larger than the thermal conductivity of the insulation that one can approximate the metal layer in the system by a lumped thermal capacity. The problem to be solved can then be stated in terms of eq. 1, subject to the following boundary and initial conditions:

1.) At the outer surface the temperature of the dewar wall is equal to that of the environment, taken to be the datum and set equal to zero, or

$$T\,(\theta,\ O) = O \tag{11}$$

2.) At the interior of the insulation the rate of heat transfer to the inner metal wall must be equal to the rate of increase in internal energy of that wall or

$$\frac{\partial T}{\partial x}\,(\theta,\ L) = -\,\frac{\rho_m\, C_m\, \delta_m}{k}\,\frac{\partial T}{\partial \theta}\,(\theta,\ L) \tag{12}$$

3.) The initial temperature distribution in the insulation is

$$T\,(O,\ x) = \frac{T_o\, x}{L} \qquad O \leq x \leq L \tag{13}$$

the temperature of the inner metal wall is uniformly T_o degrees below the datum.

By the separation of variable method it can be shown (8) that the solution to equation 13 can be written

$$T\,(O,\ x) = \left[B_n \cos(\lambda x) + C_n \sin(\lambda x) \right] e^{-\alpha \theta \lambda^2} \tag{14}$$

which reduces after applying the first boundary condition to

$$T(\theta, x) = C_n \sin(\lambda x) \; e^{-\alpha \theta \lambda^2} \tag{15}$$

The initial condition is satisfied for any value of λ which satisfies the transcendental equation

$$\cot(\lambda L) = \rho_m c_m \delta_m \frac{\alpha \lambda}{k} \tag{16}$$

The temperature distribution is, therefore, given by the relation

$$T(\theta, x) = \sum_{n=1}^{\infty} C_n \; e^{-\alpha \theta \lambda_n^2} \sin(\lambda_n x) \tag{17}$$

where λ_n are the Eigen values obtained from eq. 16. Since the boundary conditions at $x = L$ are homogeneous, the solutions in x are an orthoganal set (9) and C_n can easily be determined. Following standard methods one obtains after some rearrangement

$$C_n = \left(\frac{2 T_o}{L \lambda_n} \right) \frac{\sin(L \lambda_n) - L \lambda_n \cos(L \lambda_n)}{L \lambda_n - \sin(L \lambda_n) \cos(L \lambda_n)} \tag{18}$$

Substituting eq. 18 for C_n in eq. 17 gives finally the following relation for the temperature distribution at time θ:

$$T(\theta, x) = -2 T_o \sum_{n=1}^{\infty} \frac{\sin(L \lambda_n) - L \lambda_n \cos(L \lambda_n)}{L \lambda_n (L \lambda_n - \sin(L \lambda_n) \cos(L \lambda_n)} \cdot e^{-\frac{\alpha \theta}{L^2} (L \lambda_n)^2} \cdot \sin(\lambda_n x) \tag{19}$$

The temperature at the inner face is then obtained by setting x equal to L in eq. 19. The curves in Fig. 3 show the ratio of the temperature of the inner surface of the dewar to its initial temperature as a function of the dimensionless Fourier modulus $\alpha\theta/L^2$ with the ratio of the heat capacity of the inner metal wall, $c_m\rho_m\delta$, to the heat capacity of the insulation, $c_i\rho_i L$, as a parameter. It is apparent from an inspection of these curves that the smaller the ratio of heat capacities, the faster the tank will warm up.

Fig. 4 is derived from Fig. 3 for the purpose of calculating the time required for a liquid hydrogen dewar to warm to 90°K or to the ice point. Once the heat capacity ratio for a dewar is known, the corresponding Fourier modulus is established. A knowledge of the insulation thermal defusivity and thickness allows time to be calculated from the Fourier modulus.

Fig. 5 shows the temperature during warm up for a typical powder-insulated liquid hydrogen dewar with a 1. 0 cm thick inner metal wall and a 30. 5 cm (1 ft.) thick insulation. For this calculation it was assumed that the vacuum was broken with hydrogen or helium and that the insulation thermal conductivity is essentially that of the gas. Fig. 6 presents similar results obtained using nitrogen to break the vacuum. A comparison of Fig. 5 and 6 shows the desirability of using hydrogen or helium as the break gas. An experimental evaluation of the transient warmup of large powder insulated dewars is being conducted by the Los Alamos Scientific Laboratory. Preliminary data indicates good agreement with the curves of Fig. 5 and 6.

References

1. R. F. Blanks and K. D. Timmerhaus, "Cryogenic Engineering Advances in the Space Age", 1959 Cryogenics Engineering Conference Proceedings, pp. 3-11.

2. S. T. Stoy, "Cryogenic Insulation Development", 1959 Cryogenics Engineering Conference Proceedings, pp. 216-221.

3. E. Mayer, "Heat Flow in Composite Slabs," ARS Journal, May-June, 1952, pp. 150-158.

4. R. S. Harris, Jr., and J. R. Davidson, "An Analysis of Exact and Approximate Equations for the Temperature Distribution in an Insulated Thick Skin Subjected to Aerodynamic Heating", NASA TN D-519, January, 1961.

5. J. J. Brogan and P. J. Schneider, "Heat Conduction in a Series Composite Wall", Trans. ASME, Ser. C., Vol. 83, November, 1961, pp. 506-508.

6. H. S. Carslaw and J. C. Jaeger, Conduction of Heat in Solids, Oxford Press, 2nd ed.

7. M. Jakob, Heat Transfer, vol. 1, John Wiley and Sons, Inc., New York (1949).

8. F. Kreith, Principles of Heat Transfer, International Textbook Comp., 1959.

9. Wylie, C. R. Jr., Advanced Engineering Mathematics, McGraw-Hill Book Comp., 1951.

10. M. M. Fulk, "Evacuated powder insulation for low temperature", Progress in Cryogenics, K. Mendelssohn (ed.), Heywood and Co., London (1959).

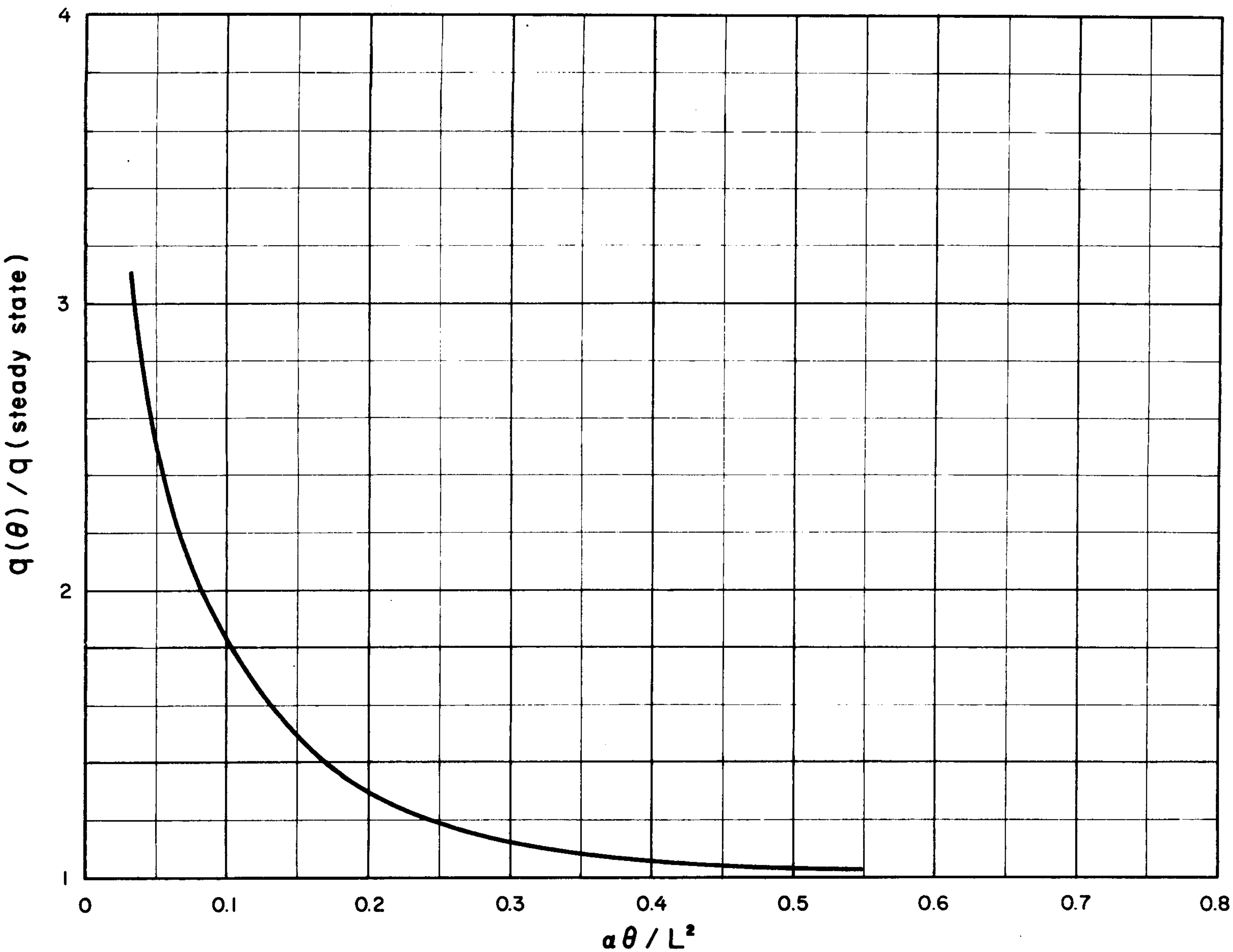

Figure 1. Dimensionless chart for calculating the cooldown of powder-insulated dewars.

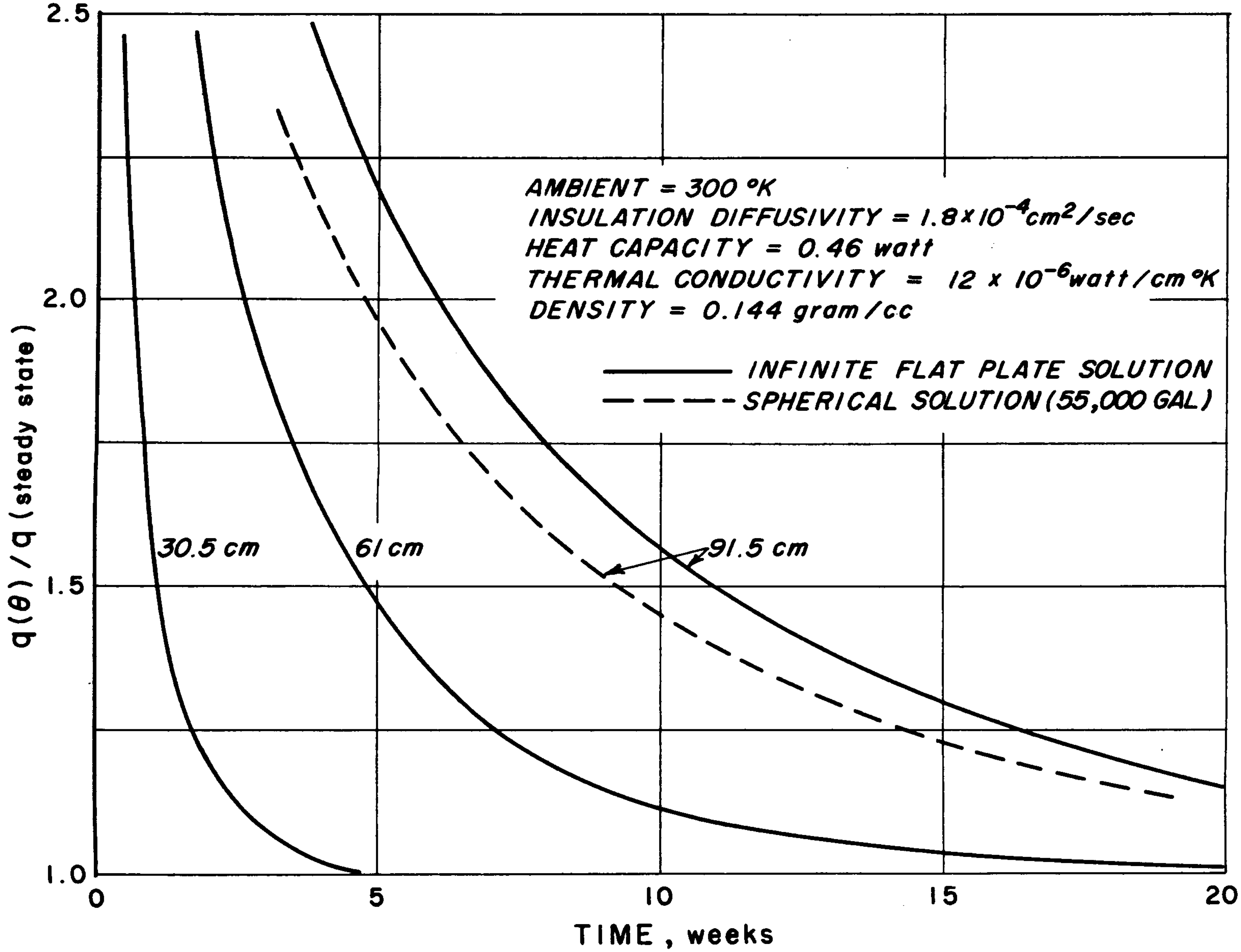

Figure 2. Typical dewar cooldown history.

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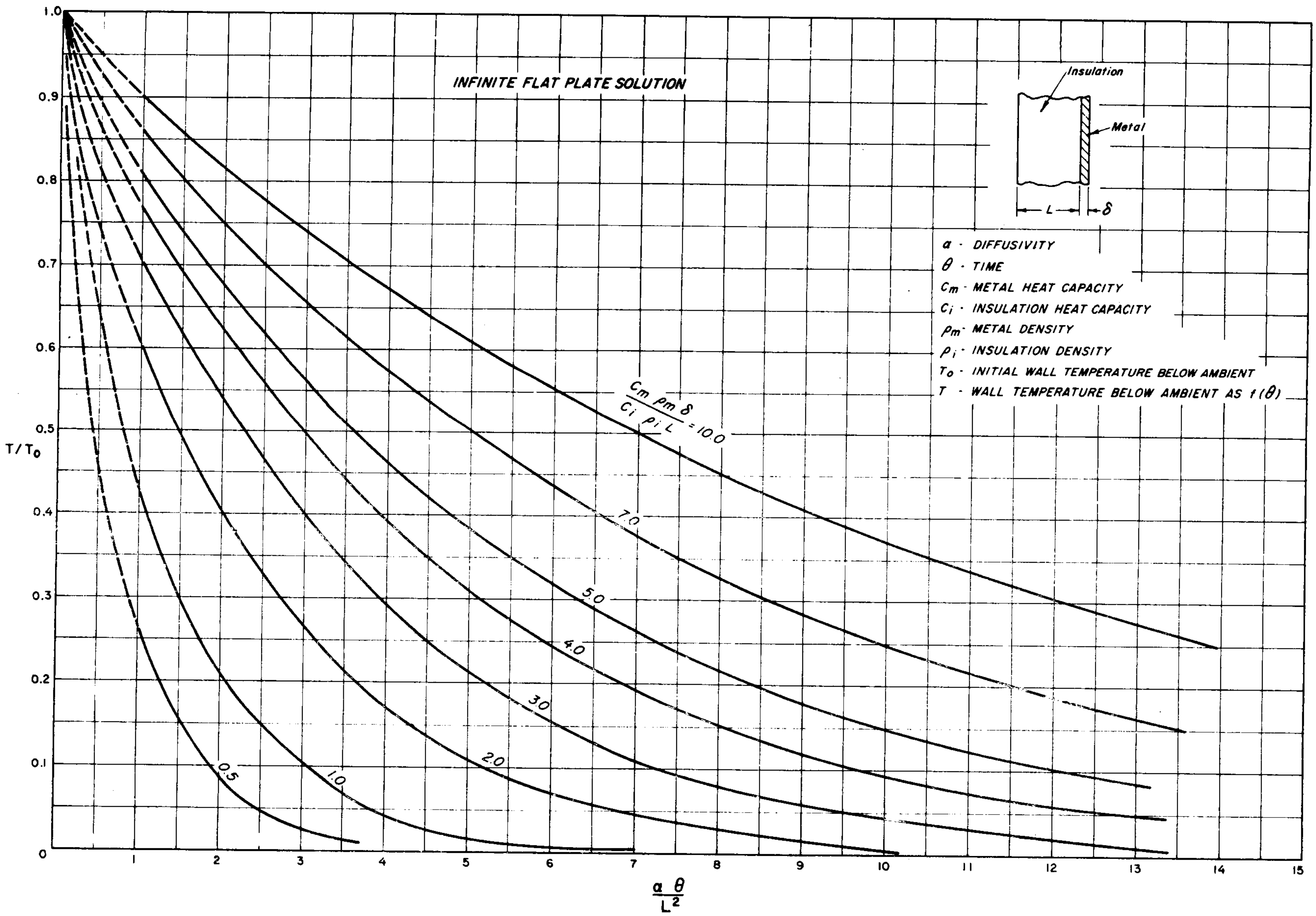

Figure 3. Dimensionless chart for calculating the warmup of powder-insulated dewars.

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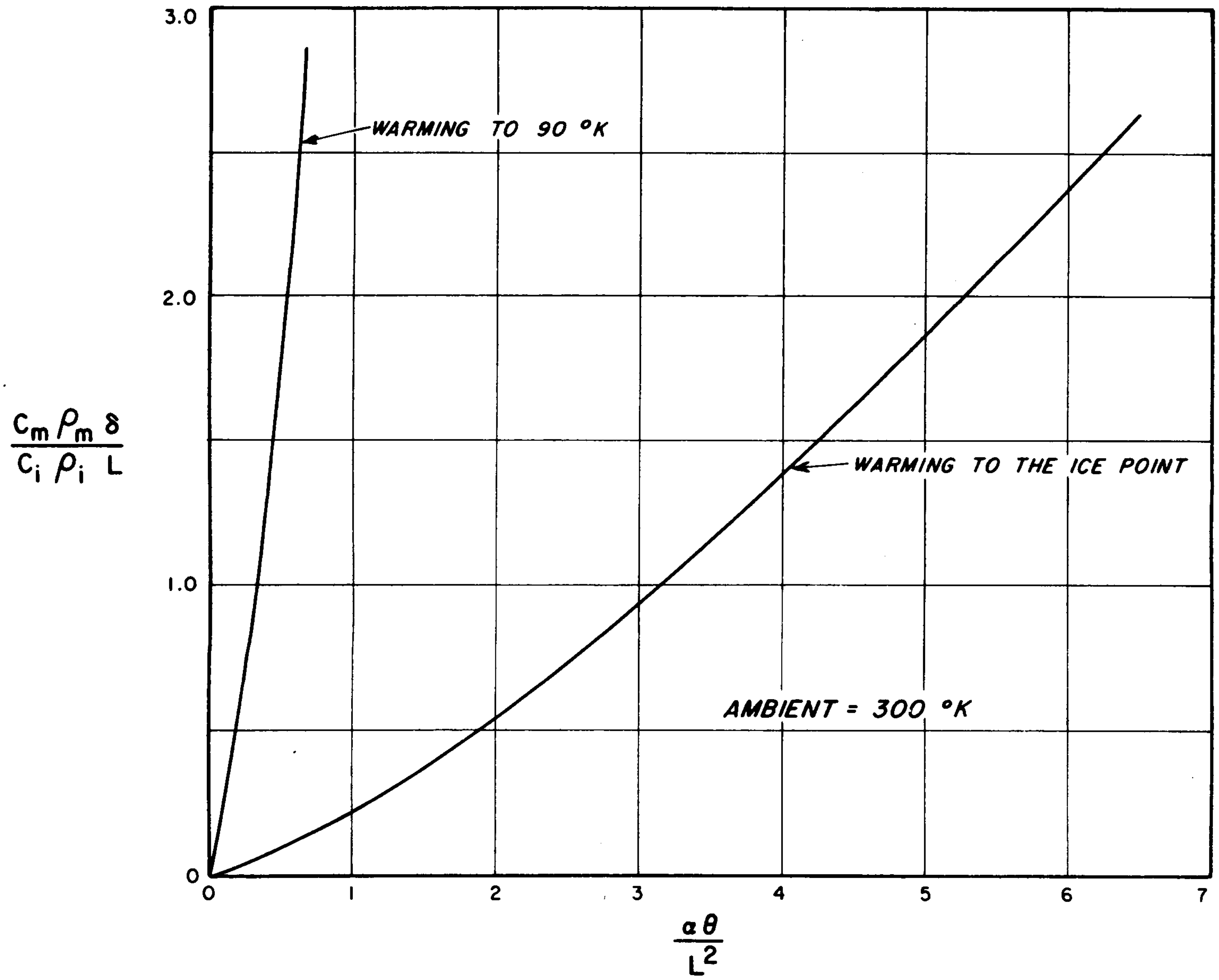

Figure 4. Dimensionless chart for calculating the warmup to specific temperature levels.

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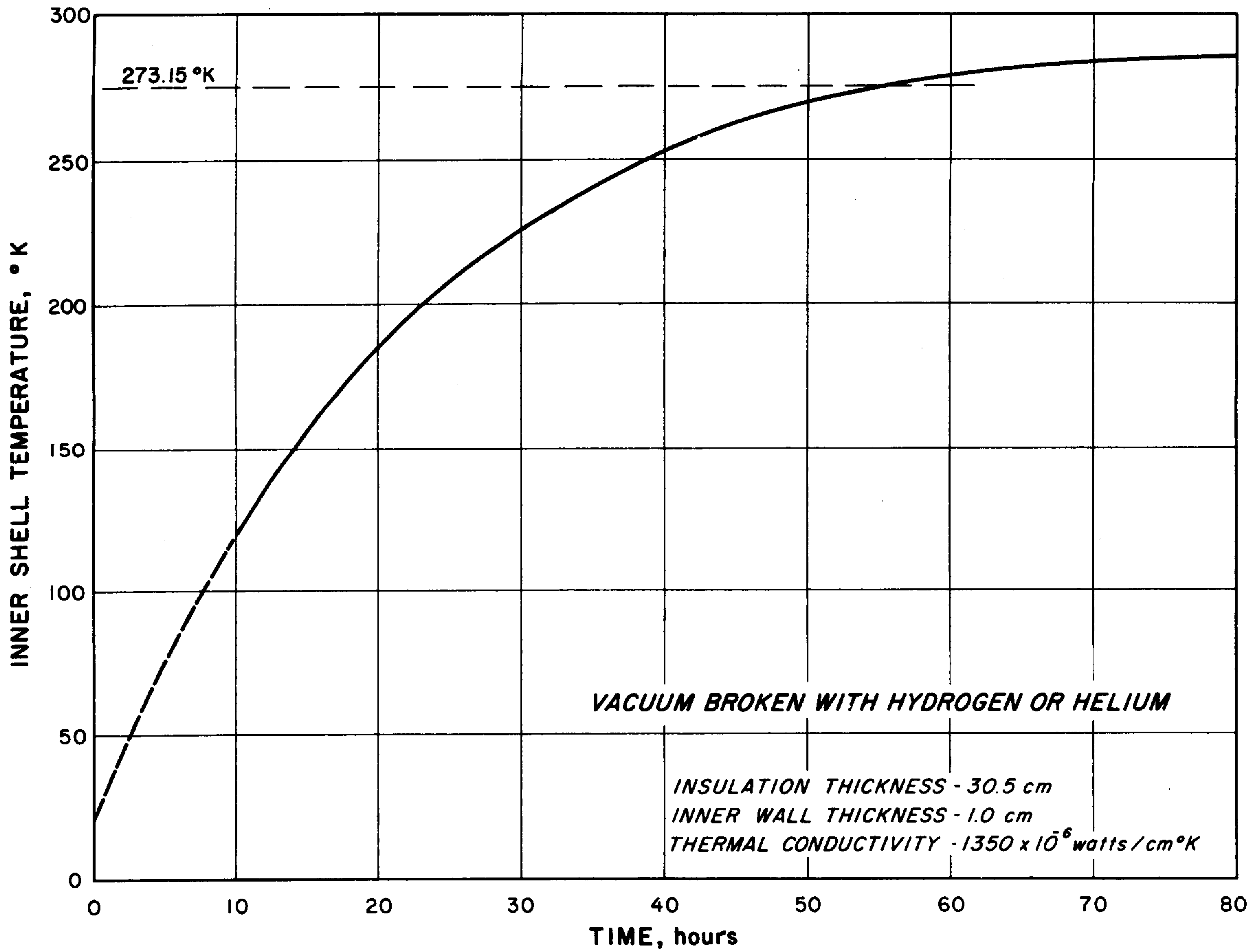

Figure 5. Warmup history with dewar vacuum broken with hydrogen or helium.

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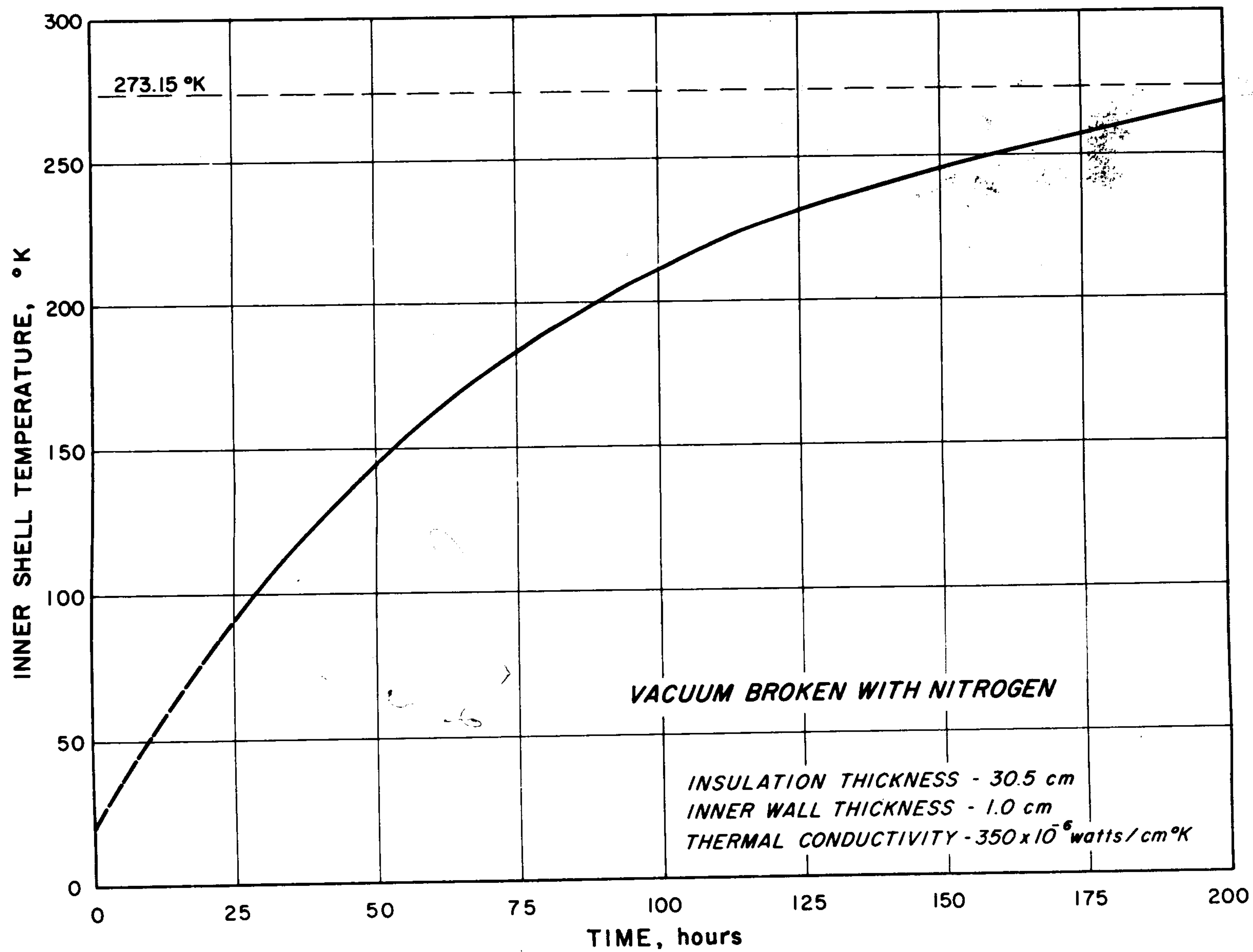

Figure 6. Warmup history with dewar vacuum broken with nitrogen.

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